THE REALM OF SCIENCE

MULTIPLICATION

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THE title of this article may strike the reader as odd, and too elementary for a university magazine. I am sure, however, that before he has got well into the subject, he will find it greater than he had any idea of, and that there are or were more methods of multiplication than the most, if not all, of us ever heard of.

To narrow down the subject, we will confine ourselves to numerical multiplication, omitting all allusion to algebraic, geometric and other kinds.

And first let us say a word about notation. This divides itself into pictorial, such as was used by the ancient Egyptians, in which numbers were represented by pictures of various objects; literal, in which letters of the alphabet were employed as in the Roman notation; and conventional, in which arbitrary symbols stood for numbers as in our present Arabic notation. The two first labored under the enormous handicap that a given picture or letter could mean only one number. Thus in Roman notation V means 5, and can never mean 50, 500, and the like. In our present system, which we by mistake call Arabic, but which the Arabians obtained from the Hindus, there are only ten figures with which any number whatever can be written. This is commonplace enough to us, but was not known in Europe before the thirteenth century. The Romans had no letter for a million, much less for anything greater, nor for anything less than one, as we have now in our decimal numbers. To express fractions, they used either what we call vulgar fractions, or more generally the Babylonian sexagesimal system, as we do yet in subdividing an hour and a degree into sixty minutes and each

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minute into sixty seconds, except that they kept on in subdividing seconds into sixty thirds, these into sixty fourths and so on, whereas we now use a mixed system and subdivide seconds decimally.

We are prone to look upon the ancients as very ignorant in the process of multiplication on account of their awkward and to us most impractical, system of numeration. Conscious of our superiority we defy anybody to multiply DCXLVIII by DCCXCIII, in competition with our multiplication of 648 by 793. This would be a rash boast and we would *certainly lose*. Why? Because the ancients used a little instrument, the abacus, which we think fit only for the kindergarten. They knew how to use it with a dexterity and a speed that our instruments, paper and pencil, cannot hope to equal. Our paper-and-pencil method is, of course, more convenient than that of having recourse to an abacus, but it is neither as rapid nor more accurate. If the reader has any doubts on the matter, he need but match his ability even in simple addition with an ordinary Japanese small tradesman. In a book entitled "Modern Instruments and Methods of Calculation—A Handbook of the Napier Tercentenary Exhibition", in 1914, Cargill G. Knott, Professor of Physics in the Imperial University of Tokyo, has a long chapter on the *Soroban* or Japanese Abacus. He shows how it is used for addition, subtraction, multiplication and division, and even for the extraction of square and cube roots.

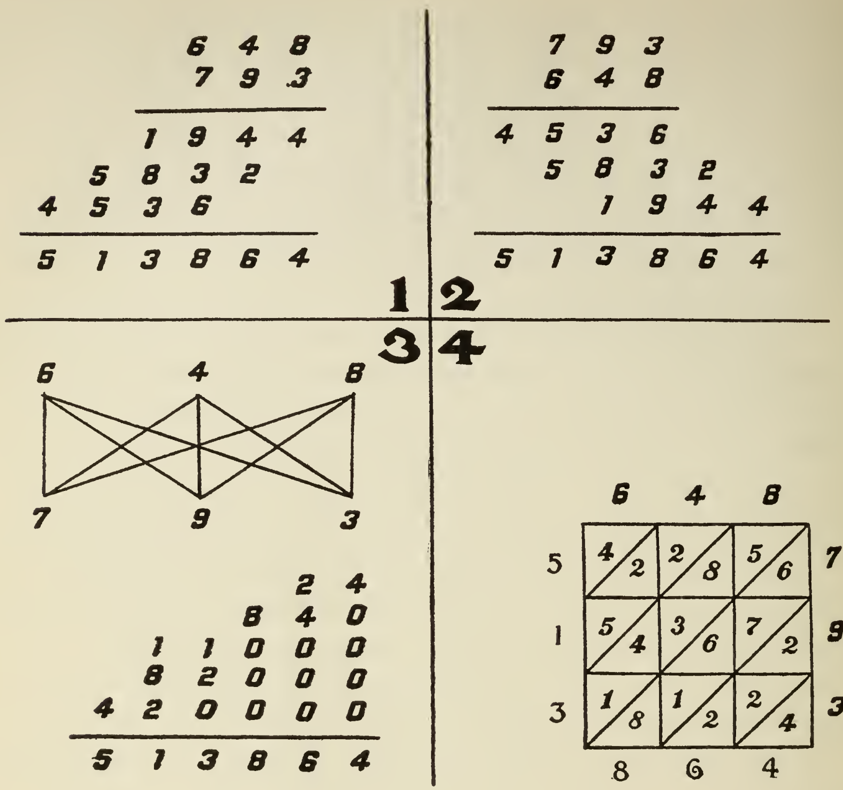
"The Soroban, or Japanese Abacus", says Knott, "may be defined as an arrangement of movable beads, which slip along fixed rods and indicate by their configurations some definite numerical quantity". There are as many rods as we like, one serving for units, another for tens, hundreds, and so forth, as well as for tenths, hundredths, and so on. There are five beads on each rod, and a sixth (sometimes a seventh) is separated from the five by a ridge. When all the beads are away from the ridge, the reading is zero. If we slide three beads on one rod towards the ridge we have 3, or 30, or 300 or 0.3, or 0.03 according to its distance from the rod we designate to hold the units place.

The single beads on the other side of the ridge are worth five times as much, and when moved towards the ridge add 5 to the indications of the other beads on the same rods. When the sum is 10 or more, the beads are set back to zero or to the excess, and one is carried to the next higher rod. Multiplication and all other operations are then performed as by us, except that the individual results are tallied at once on the abacus, and nothing at all is written.

The abacus has the great advantage that the only headwork it requires is the momentary one of the addition or multiplication of two simple numbers. Its only seeming disadvantage is that it obliterates all the intermediate figures and gives only the final answer, so that if a mistake has been made, the whole computation must be gone over again from the start. This disadvantage vanishes completely with extended use, because mistakes are far less liable to happen with the abacus, and besides, we have the identical disadvantage in our calculating machines.

Coming down more specifically to our subject of multiplication, let us now see what can be done or what has been done with our accustomed Arabic notation. Our way of multiplying 648 by 793 is to write 648 on top and 793 below it, calling the first the multiplicand and the second the multiplier. We start at the right end of the multiplier, the 3, and multiply it into each figure of the multiplicand, beginning there also at the right. We say 3 times 8 is 24, write down the 4 and retain the 2 in our head. We next multiply the second figure of the multiplicand, 4, by our 3, saying 3 times 4 is 12, but before writing down the 2 of this 12, we add to it the 2 we kept in our head and call it 14, writing down the 4, and carrying the 1 in our head. Then we say 3 times 6 is 18, add to it the 1 we carry in our head, and call it 19. We have then 648 multiplied by 3 equal to 1944. We then multiply 648 by the 9, and by the 7, shifting the product each time one place to the left because the 9 is really 90, and the 7 is 700. The three products are then added to make our complete answer, 513,864.

This sounds very elementary. True, but there is a purpose



in it. There is no erasing and no writing of unnecessary figures. But there is headwork, and that is fatiguing. Our method is the most practical with Arabic numbers. And there is no intention on the part of the writer to try and induce the reader to change it. But our present method is not and was not the only one ever used, nor is it the best in every particular.

The Hindus differed from us in two things—they exchanged our positions of the multiplicand and multiplier, and they began at the left. To multiply 648 by 793, they wrote the 793 above the 648. See number 2 in the figure. Beginning at the left, they said 7 times 6 is 42, and wrote down the 42. Then 7 times 4 is 28, writing down the 8, and adding the 2 to the 2 of the 42, getting 44, or 448 so far, by erasing the 2 of the 42 and writing the 4 in its place. With our paper-and-pencil method this erasure is awkward, but it caused no inconvenience to the Hindus who, first knew no other way, and secondly, used a white tablet strewn with red sand, in which the figures were written with a little stick. Our old custom of using a slate would then be more convenient than a paper and pencil. Then multiplying 7 times 8 and getting 56, they wrote down the 6, erased the 8 of the 448, added the 5 to it, making 13, writing down the 3, erasing the nearer 4 of 44, adding the 1 to get 5, so that the finished number would be 4,536. Then shifting the results one place to the right, they multiplied the 6 of 648 by the 9, getting 54, writing the 5 under the second figure of the previous product. Proceeding in this way and getting the three partial products, they added the results by beginning at the left, and erasing and correcting their figures whenever there was anything to carry from the next column.

While the old Hindu method seems clumsy to us, it calls for less headwork than ours, because they could stop anywhere in the midst of multiplication, whereas in our method we must by all means completely finish a partial product. They carried the numbers in their head for the least time, because they wrote them down at once. Erasures were no inconvenience to them. They had been differently educated. They lived without paper and pencil, we could not.

A third method of multiplication, called for some unknown reason *castellucio,* "by the little castle", is very much like the Hindu one, except that the multiplication of the multiplicand is begun at the right end. That is, we begin by saying, in our example, 7 times 8 is 56, write down the 6 and carry the 5, as in our present method, except that two ciphers would be added at the end, because our 7 is really 700.

A fourth method, by cross multiplication, also practiced by the Hindus, is shown graphically in our Number 3. We connect each figure of the multiplicand with each one of the multiplier. Beginning at the right (or at the left in Hindu fashion, if we like), we say 3 times 8 is 24, and write down the 24. Then 3 times 4 is 12 and 9 times 8 is 72; 12 and 72 are 84, writing down 840 under the 24, because the 4 and the 9 are 40 and 90.

Next, 3 times 6 is 18, 8 times 7 is 56, and 4 times 9 is 36, 18 and 56 and 36 are 110, with two ciphers annexed, making 11,000. Next 6 times 9 is 54 and 7 times 4 is 28, 54 and 28 are 82 with three ciphers. Lastly 6 times 7 is 42 with four ciphers. The sum of all is then 513,864. If the reader wishes to train himself to be a "lightning calculator", this method will be of great service to him. He should write only the finished product, that is, multiplying and carrying as he goes along from left to right.

A fifth method is the other extreme, and is recommended to all who find our present method of multiplication very hard. It is called *gelosia* or *graticola,* "latticed multiplication", and is illustrated in Number 4. We may begin wherever we like, after writing the multiplicand and the multiplier on the top and right side of squares ruled also diagonally in one direction. Beginning at the left, we say 7 times 6 is 42, writing the 4 in the left and the 2 in the right half of the square under the 6 and on the level of the 7. In like manner, we get 7 times 4, and all the other separate little products. When this multiplication is finished, we begin at the right lower end, for the sake of convenience and add diagonally. First we have the 4 only. Then 2 and 2 and 2 or 6 for our second figure. Next 8, 1, 6, 7, 6, or 28, with the 8 for our third figure and the 2 to carry to the next row, 1, 4, 3, 8, 5, making 23, with the 3 for our fourth figure, and the 2 to carry. And so on. This method calls for much writing, but it enables us to rest anywhere and to re-examine every step.

All this information was obtained from "A History of Elementary Mathematics", by F. Cajori, of Colorado College, an excellent little book. He treats only of arithmetic, algebra, geometry and trigonometry, and shows in a very interesting way how mathematics was developed by different nations. He emphasizes one point very clearly, that the human race progressed in its study as a child does, beginning with what we might call toys and experimental facts, and ending with abstract quantities. It would be a great mistake to make a child memorize the multiplication table before giving it blocks to play with and to arrange in rows and count up. And in every case fact preceded theory in the advancement of every science. And just as children find it hard to understand how multiplication by a fraction is really multiplication at all, since it makes the product smaller than the multiplicand, so also the greatest mathematicians required centuries of study to explain this apparent anamoly clearly. For this same reason also a teacher who sometimes gives definitions and proofs that much abler minds would condemn, but who knows how to impart the little he possesses, is a vastly more successful educator than a logically consistent and precise master. For it is better to learn a little well even if somewhat faulty, than not to learn anything. And for that matter, the most learned themselves are no better off, because there is scarcely an elementary definition of anything whatever that is absolutely pure and unobjectionable.

As this article was intended to give only a little of the various methods of multiplication, we must omit all mention of abbreviations or short cuts in our own method, such as instead of multiplying by 5 to multiply by 10 and divide by 2,, quick methods of squaring, and the like. We must also omit all reference to logarithms, generally acknowledged to be the greatest discovery ever made in computation, as also all reference to the slide rule and mechanical means of multiplication. Enough has been said however to convince the reader that there is more in such an apparently simple subject as multiplication than most of us have any idea of.